APPENDIX A

SECTION A-1. Uncertainty in measurement

Measurement is the foundation of all experimental science. Accordingly, the degree of certainty about conclusions drawn from experiments depends upon the accuracy and precision of the measurements made while conducting the experiments. The *accuracy* of a measurement indicates the amount by which it differs from a known, true value; the *precision* of a measurement is a measure of the "reproducibility" of the measurement. Let's use an analogy to make this more clear. Suppose we consider two archers who are shooting arrows at the targets below. The small circles in the centers of the targets are the bull's eyes, and the x's mark the points where the archers' arrows have struck the targets. Note that archer A shot arrows all over the place. Archer B shot arrows which landed very close together, but not near the bull's eye. Archer C shot arrows very close together too, and they also hit the desired spot! Archer B is precise, but not accurate. Archer C is both precise and accurate. Archer A should sell his bow before he hurts somebody!

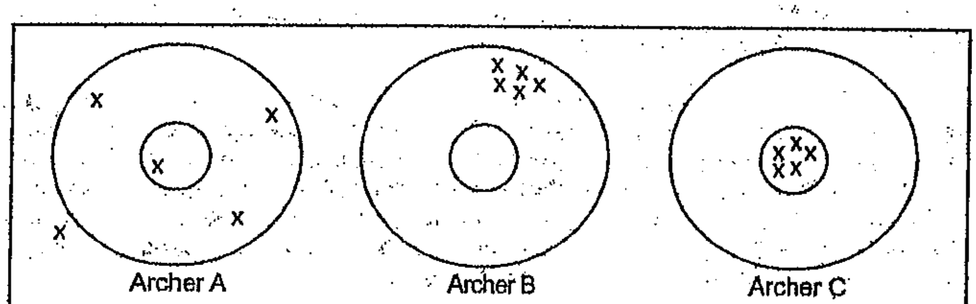

Figure A-1

Archer B is precise because he can reproduce his results. You can also think of precision as a reflection of uncertainty. If precision is high, uncertainty is low. Since archer B is precise, the uncertainty about where his arrows will hit is small. Time after time, he gets the same result. (Even though he may never hit the bull's eye.) Archer C always gets the desired result. Therefore, he is accurate. Since his accuracy is consistent, he is also precise. Archer A is not precise or accurate, poor guy.

The scientific investigator tries to be both accurate and precise. However, regardless of the exactness of the measurements, there is a degree of uncertainty. This uncertainty may be caused by the use of inadequate instruments, by inexactness in the readings made by the investigator, or by a combination of these factors. It is customary in reporting scientific measurements to include all of the figures known with certainty and one doubtful or estimated figure. For example, the metric ruler shown below in Figure A-2 can be read as 3.5 cm.

Since the 3 can be read directly from the ruler, it is known with certainty. The 0.5 cm reading is an estimate made by the observer and is only the estimate of the person reading the scale. Although estimated and therefore doubtful, the 0.5 cm is reported as part of the measurement. The digits known with certainty in any measurement and the first estimated digit are all part of the measurement and are called significant figures.

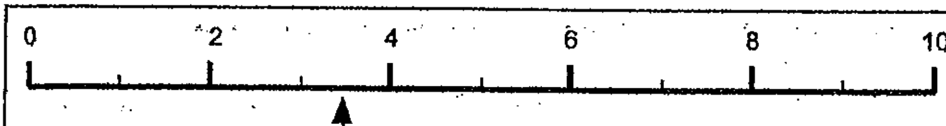

Figure A-2

It is important that a person does not report a figure such as 3.58 cm as having been read on the ruler shown above. Since the 0.5 figure is only an estimate, the next figure (0.08) cannot even be estimated and, therefore, a measurement of 3.58 indicates more information than is really known. Similarly, a reported reading of 3 cm would convey too little information, because the reader would assume that the 3 was an estimate, when in fact it is known with certainty.

A scientific measurement should include all of the information possible, but not more than can truthfully be recorded. Generally speaking, most measurements can be read to one decimal place beyond the smallest divisions on the scale being read. Thus, if an instrument is calibrated so that the closest marked divisions are at intervals of 1 cm (the ones column), readings can usually be made to the tenths column with the last digit being an estimate. Thus, we read 3.5 cm from the scale in Figure A-2.

There are two methods for expressing how much uncertainty there is in a measurement. In the first method, the uncertainty is specifically expressed. For example, the reading from the scale in Figure A-2 above was 3.5 cm. We are unsure of the 5. All we can say is that the real value is somewhere between 3 and 4. Thus the "real" value could actually fall into a range which is 0.5 cm above or below the 3.5 cm reading. We would, therefore, say that the uncertainty in the measurement is ± 0.5 cm. The measurement would be expressed as 3.5 cm ± 0.5.

The second method of expressing uncertainty involves the use of the proper number of significant figures when expressing the measurement. In this method, the amount of uncertainty is not expressed. Instead, the column in which the uncertainty exists is implied. For example, when we write the measurement as 3.5 cm, the reader will assume that the last significant figure (the 5) is an estimate. The reader then understands that the uncertainty is in the tenths column. The reader understands that the amount of uncertainty is at least ± 0.1, but it could be more (such as ± 0.2 or ± 0.3, etc.). This method of expressing uncertainty is not as specific, but it is a lot more convenient.

In measurements that have been reported correctly, <u>all nonzero numbers are always significant figures</u>. A zero may or may not be a significant figure. Why is that, and how can we tell whether a zero is significant or not? First, we should try to establish if the zero in question is serving merely to determine or locate a decimal point. When a zero serves as a determiner or indicator of a decimal point (that is, if it serves as a "place holder"), it is NOT considered to be a significant figure. For example, consider the figures shown below. In Figure A-3, note that the arrow is located between 10,000 and 20,000. As we read the scale we are, therefore, sure that the first digit in the measurement should be a 1. This is not an estimate, and it is not, therefore, the final sig fig. Now, we estimate that the second digit might be a 2. So we read 12,000 as the location of the arrow. Since the 2 is an estimate, it is the final significant digit in the measurement. We can read no more digits from the scale. However, in order to put the decimal in the proper location we need some place holders. Zeros are used for that purpose; but, <u>these zeros will not be read or estimated from the scale</u>. They are NOT significant. They serve merely as place holders. Look at Figure A-4. Note that we can be sure that the arrow is located between 10,000 and 10,001 on the scale. Thus, we are sure that the first digit should be a 1, and we are sure that the next four digits should be zeros. (We read them from the scale.) Since the arrow appears to be about half way between the two values, we estimate that the measurement should be read as 10,000.5. The 5, however, is an estimate and is, therefore, the final sig fig. The zeros in this case, are not merely place holders. They are significant!

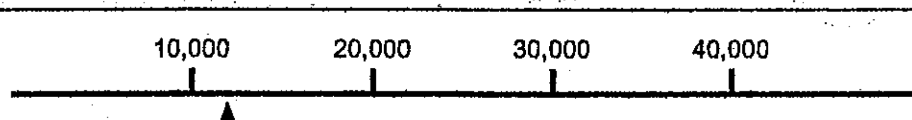

Figure A-3

Figure A-4

The following 4 rules should be observed in determining whether a digit is a significant figure. You will be expected to know these rules!

1. Nonzero digits are always significant.

2. "Leading zeros" (zeros which appear in the front portion of a number) are never significant. For example, the number 0.0039 has two "sig figs." The zeros are used to locate the decimal point. We often put a zero to the left of the decimal in numbers which have a value less than one. It is not significant.

3. "Trapped zeros" (zeros which appear between significant digits in a number) are always significant. For example, the numbers 0.0304 and 203 both have three sig figs., while the number 800006 has six sig figs.

4. "Trailing zeros" (zeros at the end of a number) may or may not be significant. They are significant only if the decimal point is expressed. If the decimal is understood (not showing), the trailing zeros are not significant. Thus, the numbers 1900. and 16.00 both contain four sig figs since the decimal point is expressed (showing) in both cases. However, in a number with an understood decimal point, the final zeros are just used to locate the decimal point and are NOT significant. Thus, the number 16,000 has two sig figs (the 6 is uncertain). If we express the decimal, 16000., the final zero is the uncertain digit and the number now has five sig figs. (Sometimes when a number ends with several zeros, the last significant one has a bar over it. For example, if the second zero in 16,000 had a bar over it, the number would then contain 4 sig figs.)

When performing arithmetic calculations involving measured values, we must express our results so that they contain only the number of significant figures justified by the uncertainty of the original measurements. Thus, it is frequently necessary to round off numbers so that a result does not appear to be more certain than the original measurements.

The following rules should be carefully observed when rounding off a measurement:

a. When the digit dropped is less than 5, the preceding digit remains unchanged; for example, 8.3734 when expressed to three sig figs becomes 8.37.

b. When the digit dropped is 5 or more, the preceding digit is increased by 1; for example, 3.6287 expressed to three sig figs becomes 3.63.

When measurements are added or subtracted, the results of the calculations should be rounded off to the column containing the leftmost uncertain digit. For example,

ADD	SUBTRACT
28.6 cm	287.56 g
327.33 cm	76.4 g
5891.212 cm	211.16 g --> (211.2 g)
6247.142 cm ---> (6247.1 cm)	

In the addition problem, since 28.6 contains the leftmost uncertain digit (the 6 in the tenths column), 6247.142 is rounded off to 6247.1. In the subtraction problem, since the 76.4 contains the leftmost uncertain digit (the 4 in the tenths column), 211.16 is rounded off to 211.2.

When measured values are multiplied or divided, count the number of sig figs in each measurement. The one which has the least number of sig figs determines how many sig figs will appear in the answer.

Example: 28 cm X 4728 cm = 132,384 cm^2 = 130,000 cm^2 (rounded)

Since 28 contains two sig figs and 4728 contains 4 sig figs, the answer must be rounded to two sig figs which would be 130,000. Since the zeros in 130,000 only indicate the position of the decimal point, they are not sig figs. 130,000 cm^2 contains only two sig figs.

Here are a couple of rules to remember:

1. When experimental quantities (measurements) are multiplied or divided, the result is rounded to the same number of sig figs as the measurement which contains the least number of sig figs.

2. When experimental quantities (measurements) are added or subtracted, the uncertain digit in the result must be in the same column as the leftmost uncertain digit in the original measurements.

This sounds confusing so look at the addition problem illustrated on page A-5. The 6 is the uncertain digit in 28.6 and it is in the tenths column. The second 3 is the uncertain digit in 327.33 and it is in the hundredths column. The second 2 is uncertain in 5891.212 and it is in the thousandth column. Of the three uncertain digits, the leftmost one is the 6 in 28.6. So, the answer should be rounded to the tenths column and becomes 6247.1. Do you now see why the answer to the subtraction problem is rounded off to the tenths column?

When doing a problem which involves both multiplication (or division) AND addition (or subtraction), you should round to the correct number of sig figs only when switching from multiplication (or division) to addition (or subtraction). Thus, you should not round off any intermediate results while you are just multiplying and/or dividing or when you are just adding and/or subtracting.

For example, try to solve the problem below with your calculator.

$$\frac{1.113\text{ cm X } 4.3\text{ cm X } 8.11\text{ cm}}{2.00\text{ cm X } 1.00\text{ cm}} - 4.5\text{ cm} + 6.32\text{ cm} = ???\text{ cm}$$

The best way to handle this calculation is to do all the multiplication and division and then correctly round the result using the proper rule. Then, do all the addition and subtraction and round that result to the correct number of sig figs using the proper rule. Finally, the two calculations can be combined and rounded to obtain the final result. The correct answer to the problem is 21 cm when rounded properly.

SECTION A-2. Problems

Problem 1. Underline any digits in the following measurements which are NOT significant.

a. 2.4421 cm	h. 42.0040 m	o. 7080.0940 mg
b. 200.41 m	i. 3000 mm	p. 0.8 mg
c. 3.00 L	j. 5.300 g	q. 674 L
d. 0.10004 cm^3	k. 00.0050050 mL	r. 4000200.080 m
e. 0.0020 m^2	l. 240 kg	s. 767003 cm
f. 00.0030030 m	m. 23,000.010 cm	t. 97600 g
g. 108,090 cm	n. 0.0060 mL	u. 8740. mg

Problem 2. Express answers to the following using the correct number of sig figs. Check your answers. You may be surprised!

a. 3.14 m X 3m =

b. 4.688 m / 2.0 m =

c. 3.4 m + 2.11 m + 0.8001 m =

d. (4.811 m)(3.1 m)(5 m) =

e. (3.4 m)(9.22 m) / 3.2 m =

f. (6.68 m^2 / 2.2 m) - 3.4 m + 7.88 m =

g. (3.42 m^2 / 2.1 m)(2.442 m / 2.10 m)(8.866 m / 2.14 m) + 4.532 m =

h. 34.5772 cm + 0.43 cm =

i. (345.2 m)(2.01m) =

j. 457.8865 mL / 3 mL =

k. 889 g - 2.886 g =

l. (45 g + 124 g) / 20. g =

m. (23,800 m)(2 m) =

3. Indicate how many sig figs are present in each of the following.

a.	671 m	_____	j.	40003 m	_____
b.	360 m	_____	k.	0.00100 m	_____
c.	059 m	_____	l.	408.0 m	_____
d.	609 m	_____	m.	20000.m	_____
e.	3040 m	_____	n.	200130 m	_____
f.	6009 m	_____	o.	20.000 m	_____
g.	3564.20 m	_____	p.	0.050440 m	_____
h.	0.042 m	_____	q.	5744 m	_____
i.	55600 m	_____	r.	99100 m	_____

4. When a number is converted into exponential form, only significant figures (digits) are used. Keeping this in mind, express the following in scientific notation.

a. 381 cm	______________________	e. 6040 L	______________________
b. 80462 kg	______________________	f. 0.003400 g	______________________
c. 0.0055 g	______________________	g. 300 mL	______________________
d. 101000 mm	______________________	h. 0.52 kg	______________________

It is important to emphasize that significant figures only exist in measurements. Therefore, the rules regarding significant figures only apply when you are working with measurements. At this point you need to learn that there are three particular cases in which the rules regarding significant figures do not apply.

First, the rules do not apply when you are working with *pure numbers*. So, for example, if we were to multiply 5 times 45, the answer would be: 5 X 45 = 225. These are pure numbers, they do not have units. Since they are not measurements, we do not round the answer using the rules for significant figures.

However, if we multiply 5 cm X 45 cm then the rules for sig figs do apply and the answer would be:

$$5 \text{ cm} \times 45 \text{ cm} = 225 \text{ cm}^2 = 200 \text{ cm}^2 \text{ (rounded)}$$

Second, the rules do not apply when you are dealing with *definitions* that relate units of measure within the same system - such as the metric system. For example, look at the problem below and calculate the answer.

$$434.5\text{mL} \times \frac{1 \text{ L}}{1000 \text{ mL}} = ???\text{ L}$$

This problem involves multiplication and division. Therefore, we count the number of sig figs in each of the measurements. The only measurement in the problem is 434.5 mL. The ratio of 1L/1000mL is a definition. Even though this ratio includes numbers with units, 1 liter is equal to 1000 mL by definition. Therefore, we look only at 434.5 mL. Seeing that it contains 4 sig figs, we round the answer to four sig figs. If a ratio is used which changes units in one system of measurement to an equivalent measure in another system of measurement, then the rules do apply. For example, calculate the answer to the problem below.

$$6764 \text{ g} \times \frac{1.00 \text{ pound}}{454 \text{ grams}} = ???\text{ pounds}$$

The ratio in the problem above converts pounds (English system) into grams (metric system). Measurements actually had to be made to determine this relationship, so the rules apply here. You cannot change pounds into grams by definition since they belong to different systems. Following the rule for multiplication and division, the answer is rounded to 3 sig figs.

Third, the rules for significant figures do not apply to *"counts,"* because "counts" are not measurements. For example, in the problem below, calculate the number of dozens of eggs we can obtain from 155 eggs:

$$8064 \text{ eggs} \times \frac{1 \text{ dozen}}{12 \text{ eggs}} = ???\text{ dozens}$$

8064 eggs is a "count." There is no uncertainty. It means exactly 8064 eggs. Similarly, there are exactly 12 eggs in a dozen. No measuring instrument is needed here, thus these cannot be measurements. The rules for sig figs do not apply. The answer is 672 dozens.

SECTION A-3. Answers to Problems

1. a. 2.4421 cm
   b. 200.41 m
   c. 3.00 L
   d. 0.10004 cm^3
   e. 0.0020 m^2
   f. 00.0030030 m
   g. 108,090 cm
   h. 42.0040 m
   i. 3000 mm
   j. 5.300 g
   k. 00.0050050 mL
   l. 240 kg
   m. 23,000.010 cm
   n. 0.0060 mL
   o. 7080.0940 mg
   p. 0.8 mg
   q. 674 L
   r. 4000200.080 m
   s. 767003 cm
   t. 97600 g
   u. 8740. mg

2. a. 9 m^2; b. 2.3 m; c. 6.3 m; d. 70 m^3; e. 9.8 m; f. 7.5 m; g. 12.3 m; h. 35.01 cm; i. 694 m^2; j. 200; k. 886 g; l. 8.5 g; m. 50,000 m^2

3. a. 3; b. 2; c. 2; d. 3; e. 3; f. 4; g. 6; h. 2; i. 3; j. 5; k. 3; l. 4; m. 1; n. 5; o. 5; p. 5; q. 4; r. 3

4. a. 3.81×10^2 cm; b. 8.0462×10^4 kg; c. 5.5×10^{-3} g; d. 1.01×10^5 mm; e. 6.04×10^3 L f. 3.400×10^{-3} g; g. 3×10^2 mL; h. 5.2×10^{-1} kg